Global and Subgroup Connectivity Maintenance for Decentralized Multi-Robot Networks under Uncertainty

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Abstract— **In this paper, we propose a decentralized uncertainty-aware bi-level optimization approach to achieve connectivity maintenance while accounting for Gaussian-distributed localization uncertainty. By integrating the theoretic approach into a graph-based approach, a minimally constraining set of connectivity edges are obtained and stay updated over the dynamically changing multi-robot communication graph. By jointly optimizing the connectivity constraints to enforce and resultant control revisions, the intended minimally revised multi-robot controllers can be obtained while ensuring safety and connectivity under uncertainty. Moreover, we introduce a fully decentralized algorithm that interleaves the constraint specification from the proposed graph-theoretic approach and solving the resulting optimization-based control problem. This enables efficient computation for large-scale systems. Simulation results demonstrate the effectiveness of our method.**

I. Introduction

Networked multi-robot systems are capable of exhibiting cooperative behaviors in various domains, e.g., search and rescue, environmental sampling and exploration, and precision agriculture. Robots within a limited communication range can achieve collective decision-making and information sharing [1] and may form subgroups to perform multiple tasks in parallel for efficient task execution [2]. For example, a group of robots can be split into subgroups to cover different terrains in disaster areas, coordinating and sharing information for effective search and rescue missions. Besides, it is also necessary for the networked multi-robot system to guarantee safety and connectivity under noisy observation, which is aroused from various estimation or prediction procedures in the real world.

Hence, to achieve reliable coordination for the networked multi-robot system, it is often necessary to 1) maintain connectivity both for all robots within the system, which is commonly referred to as maintaining *global connectivity* [3], [4], as well as the connectivity within the different subgroups, 2) guarantee the safety, i.e. robots need to avoid collision while maintaining the desired connectivity [5], and 3) ensure the scalability of the system, allowing for the efficient operation of robots in large-scale applications [6].

Existing research primarily focuses on either global or local connectivity in multi-robot systems. Global methods preserve algebraic connectivity, ensuring the positive second-smallest eigenvalue of the communication graph Laplacian [7], [8], but lack flexibility for task coordination. Local approaches [9], [10] maintain the fixed initial graph topology, often leading to conservative constraints on robot motion. To this end, a more flexible communication graph to maintain is preferred to achieve less restrictive behavior and higher task efficiency. On the other hand, task performance such as connectivity maintenance with formal guarantees also relies on tools used for constraint specification. Control barrier functions (CBF) [11] have been extensively studied for multi-robot systems to ensure safety and connectivity by constraining the control input at each time step so that the state of the system remains within a desired region [12], [13]. Our previous work [2] utilized CBFs and a graph-theoretic approach to guarantee global and subgroup connectivity with flexible multi-robot coordination. However, these approaches assume perfect localization information, and they also require synchronized communication among agents which leads to less efficient computation.

To enhance the real-time computation efficiency, distributed optimization-based frameworks, such as Consensus Alternating Direction Method of Multipliers (C-ADMM) [14], have gained interest in solving large-scale optimization problems in multi-robot collaboration. Although C-ADMM has been successfully adopted in various applications [15], [16], existing algorithms mainly focus on agent operation with *given* constraints. It is unclear how to extend such an algorithm to general multi-robot tasks with flexible connectivity maintenance, whose constraints between pairwise robots may not be explicitly defined beforehand.

We provide an intuitive illustration of our proposed method in Figure 1. The **main contributions** of this paper are three-fold. First, a novel decentralized uncertainty-aware bi-level optimization approach is proposed to achieve safety and connectivity maintenance under observation uncertainty. Probabilistic Control Barrier Certificates, such as Probabilistic Safety Barrier Certificates (PrSBC) and Probabilistic Connectivity Barrier Certificates (PrCBC), are utilized to maintain inter-robot connectivity and safety with a user-specified probability. Second, we also present a novel way to compute the optimal composition of PrCBC so that it can be directly embedded into a graph-based approach. By doing so, a minimally constraining set of connectivity edges are obtained and stay updated over the dynamically changing multi-robot communication graph. By jointly optimizing the

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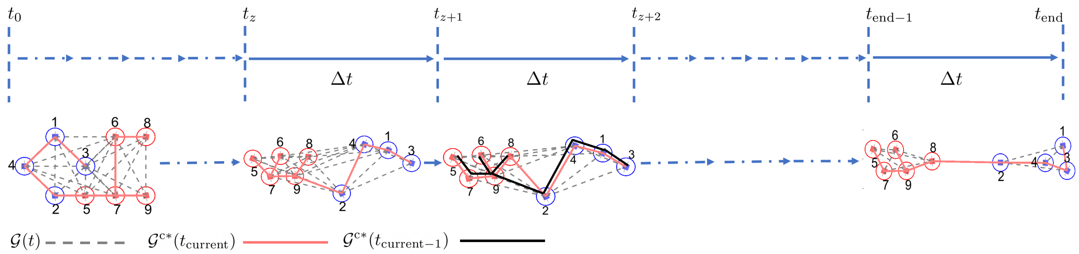


Fig. 1: Denote $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ as the communication graph of the robotic team, where each node $v \in \mathcal{V}$ represents a robot. If the Euclidean distance between the pair-wise robot i and j is smaller than the communication range, then it is assumed that the two robots are connected and can communicate with the undirected edge $(v_i, v_j) \in \mathcal{E}$ (i.e.$(v_i, v_j) \in \mathcal{E} \iff (v_j, v_i) \in \mathcal{E}$). The connectivity spanning graph can be denoted as $\mathcal{G}^{c} = (\mathcal{V}, \mathcal{E}^{c}) \subseteq \mathcal{G}$. We aim to obtain the real-time optimal connectivity graph $\mathcal{G}^{c*}(t_{\text{current}}) \subseteq \mathcal{G}(t_{\text{current}})$ to enforce at each time step for flexible multi-robot coordination under positional uncertainty, while ensuring safety and required connectivity. The enforced graph $\mathcal{G}^{c*}(t_{\text{current}})$ should be both globally and subgroup connected, meaning there is at least one path between every pair of vertices in the graph and in each induced subgroup (blue and red in this example).

connectivity constraints to enforce and resultant control revisions, the intended minimally revised multi-robot controllers can be obtained while ensuring safety and connectivity under uncertainty. Third, we introduce a Consensus Alternating Direction Method of Multipliers (C-ADMM)-based algorithm that interleaves the constraint specification from the proposed graph-theoretic approach, as well as solving the resulting optimization problem in a distributed manner. This enables efficient computation for large-scale systems.

II. Problem Statement

In this paper, we consider a robotic team S with N robots moving in a d-dimensional workspace that encompasses both free space and K static obstacles. The static obstacles[1] $o \in \mathcal{O} = \{1, ..., K\}$ are modelled as rigid sphere centred at $\mathbf{x}_o^{\text{obs}} \in \mathbb{R}^d$. The dynamics of each robot i, located at position $\mathbf{x}_i \in \mathbb{R}^d$, follow the equation $\dot{\mathbf{x}}_i = F_i(\mathbf{x}_i) + G_i(\mathbf{x}_i)\mathbf{u}_i$ with $\mathbf{u}_i \in \mathbb{R}^q$, where both $F_i : \mathbb{R}^d \to \mathbb{R}^d$ and $G_i : \mathbb{R}^d \to \mathbb{R}^{d\times q}$ are locally Lipschitz continuous functions. The positions available to the robots are affected by Gaussian-distributed noise arising from sensors, represented by $\hat{\mathbf{x}}_i = \mathbf{x}_i + \epsilon_i$, where $\epsilon_i \sim \mathcal{N}(0, \sum_i)$. We assume the robotic team $\mathcal{S}$ has been assigned M simultaneous tasks ($M \leq N$) with M divided subgroups $\mathcal{S} = \{\mathcal{S}_1, \ldots, \mathcal{S}_M\}$, where each robot i has been tasked to a subgroup $\mathcal{S}_m, \quad m = 1, \ldots, M$ with the individual task-related nominal controller $\mathbf{u}_i = \tilde{\mathbf{u}}_i \in \mathbb{R}^q$. Then given the real-time communication graph $\mathcal{G}^{c}$ determined by the observed noisy robots' locations $\hat{\mathbf{x}}_i \in \mathbb{R}^d$, the primary objective of this paper is three-fold: (i) Global and subgroup LOS connectivity of the resulting communication graph $\mathcal{G}$ remains preserved as robots move. (ii) The communication constrained robot controller $\mathbf{u}$ for all robots will be minially deviated from their nominal task-related controller $\tilde{\mathbf{u}}_i \in \mathbb{R}^q$. (iii) The proposed solution is computationally efficient and scalable, suitable for deployment in large-scale systems in a distributed fashion.

We assume that all robots are safe (i.e. $||\mathbf{x}_i - \mathbf{x}_j|| \geq R_{\text{s}}$ with R_{s} as the safety radius) and that the communication graph $\mathcal{G}^{c}$ is global and subgroup connected initially. Hence, the step-wise optimization problem can be defined as follows, boiling down to (a) identify the least constraining communication subgraph $\mathcal{G}^{c*}$ that must be maintained to preserve global and subgroup connectivity. (b) minimize the control deviation, taking into account the connectivity constraints derived from (a).

$$\mathbf{u}^* = \arg\min_{\mathcal{G}^{c}, \mathbf{u}} \sum_{i=1}^{N} ||\mathbf{u}_i - \tilde{\mathbf{u}}_i||^2 \tag{1}$$

$$\text{s.t.} \quad \mathcal{G}^{c} = (\mathcal{V}^{c}, \mathcal{E}^{c}) \subseteq \mathcal{G} \quad \text{is connected} \tag{2}$$

$$\mathcal{G}_m = \mathcal{G}^{c}[\mathcal{V}_m] \quad \text{is connected} \quad \forall m = 1, ..., M \tag{3}$$

$$\mathbf{u} \in \mathcal{S}_{\mathbf{u}}^{\sigma^{\text{s}}}(\hat{\mathbf{x}}) \cap \mathcal{S}_{\mathbf{u}}^{\sigma^{\text{obs}}}(\hat{\mathbf{x}}, \mathbf{x}^{\text{obs}}) \cap \mathcal{C}_{\mathbf{u}}^{\sigma^{\text{c}}}(\hat{\mathbf{x}}, \mathcal{G}^{c}), \quad ||\mathbf{u}_i|| \leq \alpha_i, \forall i = 1, \ldots, N \tag{4}$$

where $\mathcal{S}_{\mathbf{u}}^{\sigma^{\text{s}}}(\hat{\mathbf{x}})$, $\mathcal{S}_{\mathbf{u}}^{\sigma^{\text{obs}}}(\hat{\mathbf{x}}, \mathbf{x}^{\text{obs}})$ and $\mathcal{C}_{\mathbf{u}}^{\sigma^{\text{c}}}(\hat{\mathbf{x}}, \mathcal{G}^{c})$ are the developed admissible control space for high-probability inter-robot collision avoidance ($\mathcal{S}_{\mathbf{u}}^{\sigma^{\text{s}}}(\hat{\mathbf{x}})$), robot-obstacle collision avoidance ($\mathcal{S}_{\mathbf{u}}^{\sigma^{\text{obs}}}(\hat{\mathbf{x}}, \mathbf{x}^{\text{obs}})$), communication maintenance ($\mathcal{C}_{\mathbf{u}}^{\sigma^{\text{c}}}(\hat{\mathbf{x}}, \mathcal{G}^{c})$). α_i indicates the bounded control input for each robot. In Section III, we will introduce how to design such admissible control spaces and develop a novel distributed algorithm that **interleaves** the two processes of finding the optimal graph $\mathcal{G}^{c*}$ and solving a reformulated constrained QP problem, which is proved to produce the same solution as in the centralized version in (1).

III. Method

In our previous work [17], the concept of Probabilistic Safety Barrier Certificates (PrSBC) was introduced, constituting a probabilistic extension of the Control Barrier Function (CBF) [11]. This method ensures high-probability collision avoidance between pairwise robots, as well as robots and obstacles, by accounting for *bounded* noise (uniformly distributed) on observed robots' states [18].The Probabilistic Safety Barrier Certificates (PrSBC) have been formulated as deterministic linear control constraints, delineating the admissible control spaces These constraints guarantee safety

[1]In this paper, for simplicity, the obstacle positions $\mathbf{x}_o^{\text{obs}}$ are assumed to be known by each robot, and one can also extend our approach to consider obstacles with noisy observations as surrounding robots that are static.

with probabilities at least σ, where σ is user-define confidence level reside in the interval $(0, 1)$.

In this work[2], we relax the assumption of *bounded* system observation noise on robots' states and extend to a more general setting with Gaussian distributed observation noises that are **unbounded**, e.g. derived from Kalman filters. Hence, we propose the **Probabilistic Control Barrier Certificates** to depict the deterministic control space, rendering the system staying in the desired set with high probability σ. To be more specific, one can adopt Probabilistic Control Barrier Certificates to design the deterministic linear constraints to describe the admissible control space $\mathcal{S}_{\mathbf{u}}^{\sigma^{\mathrm{s}}}(\hat{\mathbf{x}})$ and $\mathcal{S}_{\mathbf{u}}^{\sigma^{\mathrm{obs}}}(\hat{\mathbf{x}}, \mathbf{x}^{\mathrm{obs}})$ for guaranteed pairwise robot and robot-obstacle safety with probability at least $\sigma^s, \sigma^{\mathrm{sobs}} \in (0, 1)$. Accordingly, one can also adopt Probabilistic Control Barrier Certificates to describe the deterministic admissible control space $\mathcal{C}_{\mathbf{u}}^{\sigma^{\mathrm{c}}}(\mathcal{G}^{\mathrm{slos}})$, so that with probability at least $\sigma^{\mathrm{c}} \in (0, 1)$, all pairwise communication constraints will be satisfied between robots i, j in a given spanning communication subgraph $\mathcal{G}^{\mathrm{c}} = (\mathcal{V}, \mathcal{E}^{\mathrm{c}})$ where $(v_i, v_j) \in \mathcal{E}^{\mathrm{c}}$. Thus by following the admissible control space depicted as $\mathcal{S}_{\mathbf{u}}^{\sigma^{\mathrm{s}}}(\hat{\mathbf{x}})$, $\mathcal{S}_{\mathbf{u}}^{\sigma^{\mathrm{obs}}}(\hat{\mathbf{x}}, \mathbf{x}^{\mathrm{obs}})$ and $\mathcal{C}_{\mathbf{u}}^{\sigma^{\mathrm{c}}}(\hat{\mathbf{x}}, \mathcal{G}^{\mathrm{c}})$, the moving robots are able to ensure collision-free motion while preserving the required connectivity through a given subgraph $\mathcal{G}^{\mathrm{c}}$ to maintain with prescribed high probabilities.

However, at each time step, there may exist multiple candidate subgraphs $\mathcal{G}^{\mathrm{c}}$, which satisfy both the global and subgroup connectivity requirement. To select the optimal subgraph $\mathcal{G}^{\mathrm{c}*}$ to maintain at each time step, we first propose the **Uncertainty-aware Least Constraining Tree (LCT)** algorithm. By accepting the robots' noisy positional data and nominal controllers as inputs, the algorithm is capable of identifying a specific Minimum Spanning Tree (MST) $\mathcal{G}^{\mathrm{c}*} \subseteq \mathcal{G}$. The selected edges of this MST are characterized by two attributes: 1) are least likely to break under the nominal multi-robot behaviours, and 2) satisfy the requirement of both global and subgroup connectivity. Then by maintaining such a subgraph $\mathcal{G}^{\mathrm{c}} = \mathcal{G}^{\mathrm{c}*} \subseteq \mathcal{G}$ with the associated admissible control space in (4), the problem (1) becomes a Quadratic Programming that could be directly solved in a centralized manner.

Finally, to enhance the computation efficiency, the centralized QP problem (1) can be further reformulated and solved in a decentralized form for any *given* connectivity constraints in $\mathcal{C}_{\mathbf{u}}^{\sigma^{\mathrm{c}}}(\hat{\mathbf{x}}, \mathcal{G}^{\mathrm{c}})$ using C-ADMM [14], [15]. However, the connectivity constraints in $\mathcal{C}_{\mathbf{u}}^{\sigma^{\mathrm{c}}}(\hat{\mathbf{x}}, \mathcal{G}^{\mathrm{c}})$ is unknown beforehand and will be updated as robots move. Hence, we propose the **Uncertainty-Aware Decentralized Least Constraining Tree (Dec-LCT)** algorithm that solves the bi-level optimization problem (1) with optimal $\mathcal{G}^{\mathrm{c}*}$ computed in a fully *decentralized and interleaved manner*. Each robot takes their own and neighbour's[3] noisy observation state and nominal controller information as inputs, robots iteratively updates the communication edges, constructed corresponding connectivity constraints and solve the resulting reformulated decentralized problem. Eventually, the team of robots will be guaranteed to form the optimal communication graph $\mathcal{G}^{c*}$ and the resultant optimal individual robot's controller is the solution of (1).

IV. Results

The experiment[4] is performed on a team of $N = 48$ robots, which has been divided into $M = 4$ subgroups executing M tasks respectively in parallel. Each subgroup S_m contains N_m number of robots for $m = 1, \ldots, M$. Each robot adopts the nominal controller $\tilde{\mathbf{u}}_i$ as $\tilde{\mathbf{u}}_i = -k(\mathbf{x}_i - \mathbf{c}_i)$, where $k > 0$ is a pre-assigned control gain and $\mathbf{c}_i$ is the destination position. The magenta robots rendezvous to magenta task region $\mathbf{c}_i = [1.5 \quad 0]$. The red, green, and blue robots move to $\mathbf{c}_i = [r_m \cos(\frac{2\pi i}{N_m} + \frac{2\pi t}{100 N_m}) \quad r_m \sin(\frac{2\pi i}{N_m} + \frac{2\pi t}{100 N_m})] + \mathbf{x}_m^{\mathrm{task}}$, where $r_m > 0$ is the task circle radius, $i \in S_m$, and $\mathbf{x}_m^{\mathrm{task}} = \{\mathbf{x}_1^{\mathrm{task}}, ..., \mathbf{x}_M^{\mathrm{task}}\}$ is the pre-assigned task position with t as the time step, to form a circle with radius r_m around the task position $\mathbf{x}_m^{\mathrm{task}}$. The $\mathbf{x}_m^{\mathrm{task}}$ for red, green, and blue robots in our experiment are $[-1.5 \quad 0]$,$[0 \quad -1.5]$ and $[0 \quad 1.5]$, respectively. To demonstrate the flexible robots' motion by our method, we compare our Dec-LCT against three baseline methods (Fig. 2), in which 1) only edges in the initial LCT is preserved without updating (Fig. 2d), 2) edges in the initial communication graph are preserved without updating (Fig. 2e), and 3) edges in MCCST [2] is preserved (i.e. LCT without considering uncertainty, Fig. 2f).

Fig. 2d shows that when the connectivity graph is fixed as the robot team moves, it becomes difficult for them to spread out and execute tasks, and they may easily fall into deadlock as also shown in Fig. 2e. This demonstrates the significance of our algorithm by identifying the least constraining LCT and updating it over time to maximize motion flexibility towards nominal robots' behaviors. Moreover, it is observed in Fig. 2f that the robot team will disconnect due to observation uncertainty without using our method.The corresponding numerical results in Fig. 3 show that our method ensures the best task performance with the least control perturbation, while guaranteeing safety and connectivity under localization uncertainty. Besides, it is demonstrated in Fig. 3c that our distributed algorithm achieves the similar performance as the centralized solution.

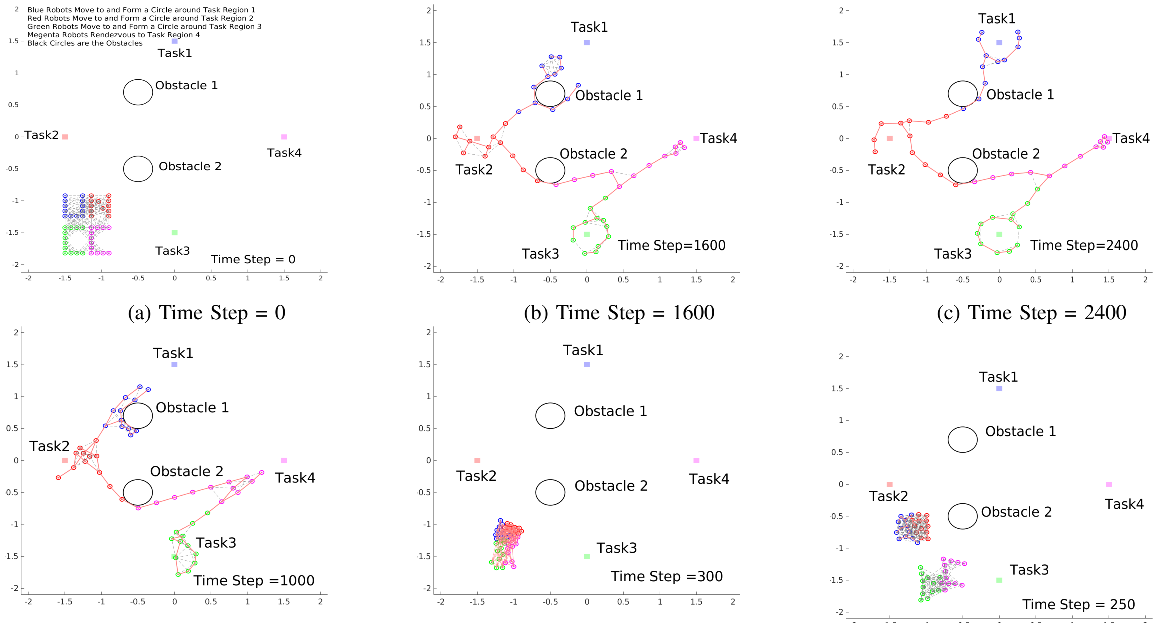
V. Conclusion

A novel distributed algorithm is presented to address global and subgroup connectivity maintenance with collision avoidance for the robotic team under uncertainty over time. Probabilistic Control Barrier Certificates are proposed to ensure a lower-bounded probability of inter-robot safety and connectivity. By integrating Probabilistic Control Barrier

[2]Due to page limit, we provide a concise explanation of our method.

[3]For our problem settings, we assume that if the robots are within the communication range, they can communicate and share information.

[4]The experiment video is available online at `https://youtu.be/sZZXkW7rQWs`. Readers are encouraged to look at the details of the experiments in the video.



(a) Time Step = 0

(b) Time Step = 1600

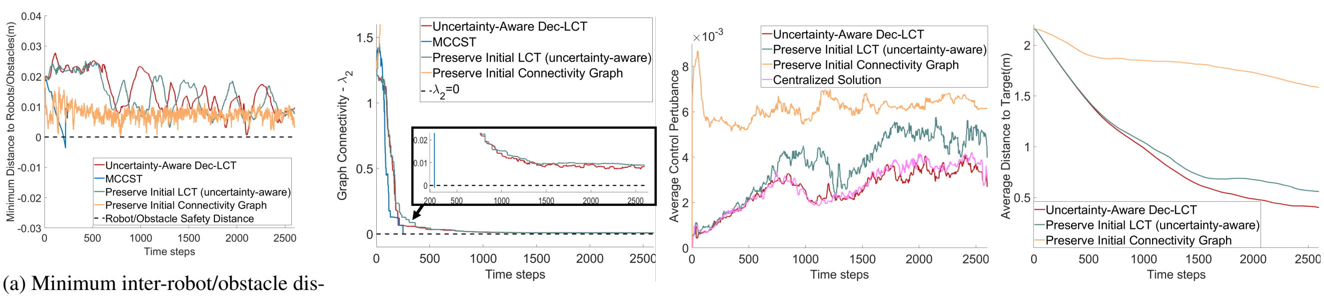
(c) Time Step = 2400

(d) Time Step = 1000(Fixed Initial LCT (uncertainty-aware), Converged)

(e) Time Step = 300 (Fixed Initial Communication Graph (uncertainty-aware), Converged)

(f) Time Step = 250 (MCCST, Disconnected)

Fig. 2: Simulation example of 48 robots are tasked to four different places. The confidence level in this experiment is set as $\sigma^{\mathrm{s}} = \sigma^{\mathrm{obs}} = \sigma^{\mathrm{c}} = 0.9$. The robot diameter in these sets of experiments is 0.03. The Multivariate Gaussian covariance matrix for measurement noise is $\begin{pmatrix} 0.003 & 0 \\ 0 & 0.004 \end{pmatrix}$. Compared to baseline methods from: initial LCT (d), the fixed initial communication graph (e) and MCCST [2](f), our proposed method can keep connectivity and enables minimally perturbed task performance due to the constraints from the connectivity maintenance on the robots.



(a) Minimum inter-robot/obstacle distance

(b) Algebraic connectivity

(c) Average control perturbation

(d) Average distance to target region

Fig. 3: Performance comparison of the simulation example in Fig. 2 w.r.t. different metrics: (a) Minimum inter-robot/obstacle distance computed by $\min\{\text{distance between robots} - 0.06, \text{distance between robot and obstacle} - 0.23\}$ at each time step. Positive meaning safety ensured (b) Algebraic connectivity evaluated by the second-smallest eigenvalue of the multi-robot Laplacian matrix. Positive meaning connectivity ensured. (c) Control perturbation computed by $\frac{1}{N}\sum_{i=1}^{N}\left\|\mathbf{u}_i^* - \tilde{\mathbf{u}}_i\right\|^2$. (d) Average distance between robots to the tasked region (the smaller, the better).

Certificates with our proposed Decentralized Least Constraining Tree (Dec-LCT) algorithm, robot teams are demonstrated to stay safe and connected under uncertainty with a minimally deviated control policy. We provide the simulation results to demonstrate the effectiveness of our method.

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